felice models

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1 WereRabbit

1.1 Circuit equations

The wererabbit neuron model is a two coupled oscillator that follows a predator-prey dynamic with a switching in the diagonal of the phaseplane. When the z in equation 1c represents the "moon phase", when ever it cross that threshold, the rabbit (prey) becomes the predator.

$$C\frac{du}{dt} = zI_{bias} - I_{n0}e^{\kappa v/U_t}[z + 26e^{-2}(0.5-u)z] - I_a \tag{1a}$$

$$C\frac{dv}{dt} = -zI_{bias} + I_{n0}e^{\kappa u/U_t}[z + 26e^{-2}(0.5-v)z] - I_a \tag{1b}$$

$$z = tanh(\rho(u-v)) \tag{1c}$$

$$I_a = \sigma I_{bias} \tag{1d}$$

$$\tag{1e}$$

Parameter	Symbol	Definition	Value
Capacitance	C	Circuit capacitance	0.1 pF
Bias current	I_{bias}	DC bias current for the fixpoint location	100 pA
Leakage current	I_{n0}	Transistor leakage current	0.129 pA
Subthreshold slope	κ	Transistor subthreshold slope factor	0.39
Thermal voltage	U_t	Thermal voltage at room temperature	25 mV
Bias scale	σ	Scaling factor for the distance between fixpoints	0.6
Steepness	ρ	Tanh steepness for the moonphase	5

Table 1: Wererabbit circuit parameters.

1.2 Abstraction

To simplify the analysis of the model for simulation purposes, we can introduce a dimensionless time variable $\tau = tI_{bias}/C$, transforming the derivate of the equations in $\frac{d}{dt} = \frac{I_{bias}}{C}\frac{d}{d\tau}$. Substituting this time transformation on equation 1

$$C\frac{I_{bias}}{C}\frac{du}{d\tau} = zI_{bias} - I_{n0}e^{\kappa v/U_t}[z + 26e^{-2}(0.5 - u)z] - \sigma I_{bias} \tag{2}$$

And dividing by I_{bias} on both sides:

$$\frac{du}{d\tau} = z - \frac{I_{n0}}{I_{bias}}e^{\kappa v/U_t}[z + 26e^{-2}(0.5 - u)z] - \sigma \tag{3}$$

Obtaining the following set of equations:

$$z = tanh(\rho(u - v)) \tag{4}$$

$$\frac{du}{dt} = z - z\alpha e^{\beta v}[1 + \gamma(0.5 - u)] - \sigma \tag{5}$$

$$\frac{dv}{dt} = -z - z\alpha e^{\beta u}[1 + \gamma(0.5 - v)] - \sigma \tag{6}$$

Parameter	**Definition**	**Value**
τ	tI_{bias}/C	–
α	I_{n0}/I_{bias}	0.0129
β	κ/U_t	15.6
γ	–	$26e^{-2}$
ρ	Tanh steepness for the moonphase	5
σ	Scaling factor for the distance between fixpoints	0.6

Table 2: Wererabbit abstract parameters mapping.

2 Boomerang

$$z = tanh(\rho(v - u)) \tag{7}$$

$$\frac{du}{dt} = 1 - \alpha e^{\beta v}[1 - \gamma(0.3 - u)] - \sigma z \tag{8}$$

$$\frac{dv}{dt} = -1 + \alpha e^{\beta u}[1 + \gamma(0.3 - v)] + \sigma z \tag{9}$$

3 Nagini

This neuron tries to implement the FitzHugh-Nagumo model. Additionally, in order to set the circuit in the right operating point, it relies on a variant of the nullclines technique.

Parameter	**Definition**	**Value**
τ	tI_{bias}/C	–
α	I_{n0}/I_{bias}	0.0129
β	κ/U_t	15.6
γ	–	$26e^{-2}$
ρ	Tanh steepness for the moonphase	5
σ	Scaling factor for the distance between fixpoints	0.6

Table 3: Boomerang abstract parameters mapping.

3.1 Overview of the FitzHugh-Nagumo equations

According to the FitzHugh-Nagumo model, the neuron behaviour can be described by the differential equations (10).[1]

$$\dot{v} = v - \frac{v^3}{3} - w + I_{app} \tag{10a}$$

$$\tau\dot{w} = v + a - bw \tag{10b}$$

Where v is the membrane potential —in the implemented circuit, this will be the voltage in the main capacitor—; w is a slow variable of the model, which is in charge of returning the system to its initial resting state; I_{app} is the current coming from a synapse; a and b are constants; and τ is the time constant of the neuron.

The objective of the implemented circuit will be to replicate this equations, but using electric components.

3.2 Circuit equations

The circuit proposed in [1], to model the membrane potential, is the one in figure 1.

The circuit in figure 1 can be described by equation (11). There, V is the membrane potential; I_{app} is the current going into the neuron; $I_{passive}$, I_{fast} and I_{slow} are the currents through the passive element, and the corresponding current sources; τ_{fast} and τ_{slow} are the time constants for each current source; V_{fast} and V_{slow} are current sources's internal variables; δ_{fast} and δ_{slow} are parameters that set the voltage range in which the current sources are active; g_{max} is a parameter from the circuit —we aware that for the fast component it is positive and for the slow it is negative—; and Erev is the reference voltage for the circuit, in our case, GND (0 volts).

[1] This is a particular case of the complete FitzHugh-Nagumo model, but as the complete model is out of the scope of this document, this simplified version is preferred.

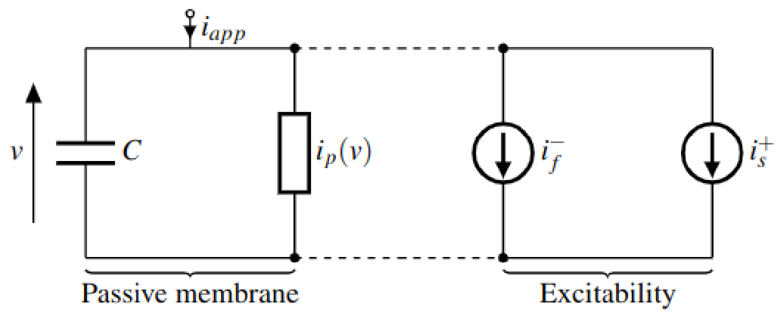

Figure 1: Ideal circuit implemented

$$C\frac{dV}{dt} = I_{app} - I_{pasive} - I_{fast} - I_{slow} \tag{11a}$$
$$\tau_{fast}\frac{dV}{dt} = V - V_{fast} \tag{11b}$$
$$\tau_{slow}\frac{dV}{dt} = V - V_{slow} \tag{11c}$$
$$I_{pasive} = g_{max}(V - E_{rev}) \tag{11d}$$
$$I_{fast} = \alpha_{fast}\tanh(V_{fast} - \delta_{fast}) \tag{11e}$$
$$I_{slow} = \alpha_{slow}\tanh(V_{slow} - \delta_{slow}) \tag{11f}$$

These are the equations implemented by the circuit, and the ones you can use to model the neuron's behaviour.

3.3 Nullclines technique and its applied variant

The nullclines technique, applied to the FitzHugh-Nagumo model, implies to put the variable v in the x axis of a plot, and variable w in the y one. Then, to draw the geometric locus formed by $\dot{v} = 0$ and $\dot{w} = 0$. This gives a line with positive slope plot for the w nullcline, and a И shape for the v nullcline.

When proposing this circuit to implement the FitzHigh-Nagumo neuron, [2] adapts the nullcline technique, but to work with custom I-V curves for the circuit. They define fast and slow currents (I) curves, then they plot them against the membrane potential (V). I_{fast} and I_{slow} are defined in (12), where I_{pas}, I_{fast} and I_{slow} are the currents passing through the passive, fast and slow elements; how to implement these elements is described in detail in the following section.

$$I_{fast} = I_{pas} + I_{fast} \tag{12a}$$
$$I_{slow} = I_{pas} + I_{slow} + I_{fast} \tag{12b}$$

When plotting I_{fast} versus V (membrane potential), the plot should have an N shape, while the I_{slow} versus V should be a line with positive slope. The neuron should spike when the operating point is in the N shape's negative slope region. For further information of how to set up the operating point of the circuit, you can look at [2], [3].

4 Snowball

5 Bucket Neuron

$$\frac{dV}{dt} = -\frac{V}{\tau} + \frac{I}{C(V)} \tag{13}$$

$$C(V) = \begin{cases} \text{const}, & \text{linear} \\ \sqrt{1/V}, & \text{nonlinear, depletion/weak} - \text{inversion} \\ \exp(V), & \text{nonlinear, accumulation/strong} - \text{inversion} \end{cases} \tag{14}$$

- Linear
- Non-linear $V \propto 1/\sqrt{-q}$
- Leakage optional (probably clocked)

Parameter	**Definition**	**Value**
τ	Leakage time constant	x ms
C(V)	Membrane capacitance	y fF

Table 4: Bucket neuron parameters

References

[1] L. Ribar, "Synthesis of neuromorphic circuits with neuromodulatory properties," Ph.D. dissertation, 2020.

[2] L. Ribar and R. Sepulchre, "Neuromorphic control: Designing multiscale mixed-feedback systems," *IEEE Control Systems Magazine*, vol. 41, no. 6, pp. 34–63, 2021.

[3] L. Ribar and R. Sepulchre, “Neuromodulation of neuromorphic circuits,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 8, pp. 3028–3040, 2019.